What is the time of particle production in hadronic collisions?

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Abstract

Formation time of final state particles in hadronic collisions is studied in a very simple model which contains the formation time as a free parameter to be determined by comparison of calculated Bose-Einstein correlation functions with the available data. Final state pions are either products of resonance decays or are "directly" produced. The "direct" production is simulated by an immediate decay of a resonance. For "direct pions" forming about a half of final state pions and for formation times of resonances within the interval 0.2-0.4 fm/c we get density of sources which leads to Bose-Einstein correlations of two identical pions consistent with recent data. The formation time of 0.2 to 0.4 fm/c is shorter then expected and it may have consequences for construction of models of proton-nucleus and nucleus-nucleus interactions. Presented at the Workshop on Heavy Ion Physics, Bratislava, Sept. 2nd-6th,

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1 Introduction

Details of dynamics of hadronic collisions in the region of a few hundred GeV are not yet completely understood, since a large part of the process can not be described by Perturbative Quantum Chromodynamics (PQCD). One of the most important parameters characterizing the process is the formation time of secondary hadrons. It has been introduced in different formulations and studied in the context of different dynamical models [1-15]. The formation time is of particular importance in studies of proton - nucleus (pA) and nucleus - nucleus (AB) collisions since it imposes constraints on the evolution of cascades and in this way it directly influences energy densities which can be reached in heavy - ion collisions [15]. The main observation made in the present paper is based on the fact that the formation time - in a way which is unfortunately not completely model independent - can be determined from Bose - Einstein correlations of identical particles in pp or πp collisions. For larger formation times the system expands longer prior to emitting final state stable and unstable hadrons and due to a longer expansion, identical pions with close momenta are emitted from larger relative distances. In our model of hadronic production the formation time enters as a free parameter which can be determined from the correlation function $C(k_1, k_2)$ for two identical pions with four - momenta k_1, k_2. The purpose of the present paper is to determine the formation time τ_f in a simple model of production of secondary particles in hadronic collisions. We have to treat inevitably the question of the influence of resonance decays on the correlation function and we have to make use of the data on the Bose - Einstein correlations of identical pions. We shall use recent data of EHS/NA-22 collaboration [16]. The paper is organized as follows: In the next Section we shall present our simple model of multiparticle production and we shall analyze the role of resonance production on Bose - Einstein correlations of identical pions. In Sect.3 we shall compute correlation function of identical pions,compare it with available data and estimate the formation time of hadrons. Sect.4 contains discussion on the influence of the value of formation on the evolution of pA and AB interactions. Comments and conclusions are presented in Sect.5.

2 A simple model of Bose - Einstein correlations of identical pions in hadronic collisions

In HBT studies of interferometry of identical particles the correlation function $C_2(k_1, k_2)$ is expressed as a square of Fourier Transform (FT) of the density of sources

$$C_2(q, K) \equiv C_2(k_1, k_2) = \left| \int \rho(x, K) e^{iq.x} d^4x \right|^2 \tag{1}$$

where four-vectors q, K are defined as

$$q = k_1 - k_2, \qquad K = \frac{k_1 + k_2}{2} \tag{2}$$

and the density of sources (the Wigner distribution) is normalized to 1 by

$$\int \rho(x, K) d^4x = 1 \tag{3}$$

We shall study $C_2(\vec{q}, K)$ as function of the longitudinal momentum $q_z \equiv q$ only. For that purpose we put into Eq.(1) $\vec{q}_T = 0, q_0 = 0$ and get

$$C_2(q) = |\int \rho(z, K)e^{iqz}dz|^2 \tag{4}$$

where $\rho(z, K)$ is $\rho(z, t, K)$ integrated over time. Our density distribution is independent of x,y since in our model resonances move only along the z-axis and therefore they decay with x = y = 0.

In hadronic collisions about a half of final state pions appear as products of resonance decays. A resonance has a formation time τ_f and a mean life-time τ_d in its rest frame and both these times are Lorentz dilated. Depending on its rapidity, resonance travels some distance before decaying. Two identical pions originated by decays of two different resonances may have close momenta and be produced from two distant sources. This leads via Bose- Einstein interferometry to an increase of $C_2(q, K)$ for small values of q. We shall show that a superposition of resonances and of directly produced pions gives the two body correlation function $C_2(q)$, which is consistent with data.

The model we are studying is admittedly oversimplified, the most drastic assumption consists in putting transverse momenta of resonances equal to zero.These simplifications permit us to do most of calculations by hand and keep the discussion as transparent as possible. In our opinion such an approach permits to get an insight into the problem and in this aspect it is complementary to less transparent Monte Carlo computations.

A large amount of models of hadronization in e^+e^-, ep and hadron- hadron collisions has been proposed, some of them can be traced back from Refs. [18-21]. In most of these models an intermediate partonic stage is followed by cluster formation and decay. It is not clear whether there are some intermediate "heavy clusters" which decay after some time to known hadronic resonances. Since we wish to have the model as simple as possible we shall not discuss such intermediate stages and we shall only assume that well known hadronic resonances are formed after a common formation time τ_f and after being formed they decay according to schemes known from experiment. The value of the formation time τ_f will be considered as a free parameter. Studies of resonance production in pp collisions have shown that about a half of final state pions comes from decays of well known hadronic resonances, although there exist also estimates that this fraction is larger. Final state pions which cannot be ascribed to decays of known resonances are referred to as being "directly" produced. It is possible that a part of these pions is due to decays of rather broad resonances. In our simplified model we describe "directly" produced pions as decay products of a resonance with vanishing life-time. Direct pions are thus produced rather early and not far from the point of the hadronic collision. The influence of resonance production on spectra of their decay products has been studied in detail [22] and literature on the effects of resonance decays on HBT interferometry can be traced back from Ref.[23].

We shall present here a very simplified and transparent model. In this model we assume that in a hadronic collision:

i) Resonances are formed in a time τ_f after the collision. The value of τ_f is a free parameter of our model.

ii) After being formed a resonance decays with the mean life-time τ_d, taken from experiment.Both τ_f and τ_d are Lorentz dilated by $\gamma = (1-v^2)^{-1/2}$ where v is the velocity of the resonance.

iii) Transverse momentum of resonances vanishes, their velocities have only components along the axis of collision (z-axis). This assumption makes the model somewhat unrealistic, but simplifies calculations and makes the model rather transparent.

iv) A part of pions is produced "directly". The direct production is described as a decay of a resonance with a vanishing mean life-time.

v) We shall work in the cms of hadronic collision and consider only simple kinematical situations in which the momentum $\vec{K} = (\vec{k}_1 + \vec{k}_2)/2$ is small and perpendicular to the axis of the collision (z-axis) and the momentum $\vec{q} = \vec{k}_1 - \vec{k}_2$ is parallel to the z-axis. This corresponds to $y_{c.m.} \approx 0$ and K_T small.

We shall now study the behaviour of the correlation function $C_2(q,K)$ of two identical pions caused by resonance decays. The two interfering amplitudes are shown in Fig.1. We assume that the two pions have - in the simple situation considered - the same energy, therefore $q_0 = k_{10} - k_{20} = 0$. We shall start with calculating function $\rho(z,K)$ for a particular resonance, then we shall sum over resonance contributions and take the Fourier Transform as shown in Eq.(1).

Width , of a resonance of mass M, decaying to two particles of mass m is given in the resonance rest frame as

$$, = \int |T|^2 \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \delta(\vec{p}_1 + \vec{p}_2)\delta(M - E_1 - E_2) \tag{5}$$

where the standard and self-explanatory notation has been used. Making use of $E_1 = E_2 = m_T ch(y)$ we can rewrite Eq.(4) for the decay to two equal mass particles as

$$, = \int |T|^2 \frac{d\phi p_T dp_T dy}{(2\pi)^6 2M\sqrt{M^2 - 4m_T^2}} [\delta(y - y_1) + \delta(y - y_2)] \tag{6}$$

where $m_T^2 = m^2 + k_T^2$ and

$$y_{1,2} = ln\left((M/2m_T) \pm \sqrt{(M/2m_T)^2 - 1}\right); \quad y_1 = -y_2 \tag{7}$$

Boosting the resonance to rapidity y_R and normalizing the decay probability to 1, with $|T|^2$ held constant we get

$$\frac{dP}{p_T dp_T d\phi dy} = \frac{1}{\pi}\frac{1}{\sqrt{M^2 - 4m_T^2}}[\delta(y - y_R - y_1) + \delta(y - y_R + y_1)] \tag{8}$$

This probability distribution is normalized as

$$\int \frac{dP}{p_T dp_T d\phi dy} p_T dp_T d\phi dy = 1 \tag{9}$$

Note that in order to keep the calculations simple we are using here and in what follows a "zero width approximation" for distribution of resonance masses.

Eqs.(6) and (7) show that resonance products are shifted in rapidity by $\Delta y = \pm y_1$ with respect to the rapidity of the resonance. The value of this shift may be rather large. For instance for decay of the ρ- meson to two pions with $p_T \approx 0$ we get $\Delta y \equiv y_1 \approx 1.5$. A pion with $y \approx 0$ and $p_T \approx 0$ is thus produced by a ρ with $y_R \approx \pm 1.5$. Such a ρ moves with velocity $v \approx tanh(y_1)$ in the rest frame of the pion.Note that for larger values of p_T of the pion the rapidity difference between the pion and the ρ becomes smaller and for $p_T^2 + m^2 = m_\rho^2/4$ the rapidity difference vanishes.

A ρ with rapidity y_1 needs some time for its formation and some time for its decay. Pion with $y \approx 0$ and $p_T \approx 0$ is thus emitted some distance away from the origin. Two identical pions, both with small y and p_T and originated in decays of two different ρ's come thus from two distant sources as shown in Fig.1.

For resonance decays to two unequal mass particles $M \to m_1 + m_2$ Eqs. (5) -(8) are somewhat modified.Calculation is straightforward, the final result being

$$\frac{dP}{d\phi p_T dp_T dy} = \frac{1}{4\pi\sqrt{k^2 - p_T^2}}[\delta(y - y_R + y_1) + \delta(y - y_R - y_1)] \tag{10}$$

where

$$k \equiv (p_T)_{max} = \frac{1}{2M}[M^2 - (m_1 + m_2)^2]^{1/2}[M^2 - (m_1 - m_2)^2]^{1/2} \tag{11}$$

and

$$y_1 = ln(\alpha + \sqrt{\alpha^2 - 1}), \qquad \alpha = \frac{M^2 - (m_2^2 - m_1^2)}{2m_1 M} \tag{12}$$

This equation is valid for vanishing transverse momenta of decay products. For $p_T \neq 0$ masses m_1 and m_2 in Eq.(12) should be replaced by the corresponding transverse masses.

Expressing the four-vector K in Eq.(2) in terms of y, p_T, ϕ the density of points in which pions are produced $\rho(z, t; K)$ can be written as follows

$$\rho(z, t; y, p_T, \phi) = \sum_R \int P(z, t; y_R).\frac{dn_R}{dy_R}.\frac{dP}{p_T dp_T d\phi dy} dy_R \tag{13}$$

Here dn_R/dy_R is the rapidity density of the resonance R, $dP/p_T dp_T d\phi dy$ is given by Eq.(10) and $P(z, t; y_R)$ is the probability density that resonance R with rapidity y_R decays in the space-time point (z, t). Since we have assumed that resonances move along the z-axis, coordinates x,y of the position of resonance decay vanish. It follows from Eq.(13) that the correlation $C_2(q, K)$ is essentially given by the probability distribution $P(z, t; y_R)$. For the case of $y = 0$ which we consider here, the function $P(z, t; y_R)$ is symmetric with respect to $z \to -z$ and we shall calculate it only for $z \geq 0$. In this case out of two δ-functions in Eq.(8) only the one with $y_R = y_1$ contributes.

The function $P(z, t; y_R)$ is given by the space-time features of formation and decay of resonance R. There are many models of formation of final state hadrons in hadronic collisions. To keep our model as simple as possible we shall select a particularly simple version. We assume that a resonance is formed in its rest frame in time τ_f and in this frame the probability of resonance being already formed at time τ is

$$P_f(\tau) = 1 - exp(-\tau/\tau_f) \tag{14}$$

In the frame in which resonance R has rapidity y_R its velocity is $v(y_R) = tanh(y_R)$, the formation time is dilated to $t = cosh(y_R)\tau_f$ and the distance travelled by R is $z = v(y_R)t = sinh(y_R)\tau_f$.Probability that resonance R is already formed at the distance z from the origin becomes

$$P_f(z) = 1 - exp(-z/z_f), \qquad z_f = sinh(y_R)\tau_f \tag{15}$$

The resonance is formed within the interval$(z, z + dz)$ with probability density

$$\rho_f(z) = \frac{dP_f(z)}{dz} = \frac{1}{z_f}e^{-z/z_f} \tag{16}$$

Assuming a standard exponential decay law, the probability density for decay in the interval $(z, z + dz)$ of resonance produced in z_1 is

$$\rho_d(z) = \frac{1}{z_d}exp[-(z - z_1)/z_d]; \quad z_d = v(y_R)t_d = sinh(y_R)\tau_d$$

where τ_d is the decay time in the rest frame of the resonance. Probability density $P(z, t; y_R)$ in Eq.(13) is then given as (t suppressed)

$$P(z; y_R) = \int_0^z \rho_f(z_1)\rho_d(z - z_1)dz_1 = \frac{1}{z_f - z_d}[e^{-z/z_f} - e^{-z/z_d}] \tag{17}$$

where

$$z_f = sinh(y_R)\tau_f, \qquad z_d = sinh(y_R)\tau_d$$

Note that in our model the z and t of resonance decay are stronly correlated

$$P(z, t; y_R) = P(z; y_R)\delta(t - \frac{z}{v(y_R)})$$

and $P(z; y_R)$ is $P(z, t; y_R)$ as mentioned below Eq.(4).

It is easy to see that $P(z; y_R)$ satisfies the consistency criteria: (i) Integral from 0 to ∞ of $P(z; y_R)$ is equal to 1,(ii) for $z_f \to 0$ particles are formed immediately and $P(z; y_R)$ approaches $(1/z_d)exp(-z/z_d)$ as expected, (iii) for $z_d \to 0$ particles decay immediately and $P(z; y_R)$ approaches $(1/z_f)exp(-z/z_f)$ as it should.

Function $P(z; y_R)$ for negative z is given as $P(z; y_R) = P(-z; y_R)$. According to Eq.(1) the correlation function is expressed in terms of the Fourier transform of $\rho(z; K)$. As seen from Eq.(13) the z-dependence is given only by $P(z; y_R)$. Note that we consider two pions of equal energy but different longitudinal momenta.In such a situation the time of resonance decay does not enter the results. We shall therefore need the Fourier transform (FT in what follows) $\tilde{P}(q; y_R)$ defined as follows

$$\tilde{P}(q; y_R) = \int_{-\infty}^{\infty} dze^{iqz}P(z; y_R) \tag{18}$$

Inserting Eq.(17) into Eq.(18) we get

$$\tilde{P}(q; y_R) = \frac{1 - z_f z_d q^2}{[1 + (z_f q)^2][1 + (z_d q)^2]} \tag{19}$$

where $\tilde{P}(q; y_R)$ is normalized by $\tilde{P}(q = 0; y_R) = 1$. The final expression is obtained by Eqs.(1),(13) and (19), inserting branching ratio BR(R) for the decay of resonance R to a pion of given type:

$$|P(q)|^2 \equiv C_2(q, K) = \left| \frac{\sum_R \tilde{P}(q; y_R) w_R(K)}{\sum_R w_R(K)} \right|^2 \tag{20}$$

where $\tilde{P}(q; y_R)$ is given by Eq.(19), $w_R(K)$ is obtained via Eqs.(8) and (13)

$$w_R(K) = \tilde{f}_R(K).\frac{dn_R}{dy}.BR(R) \tag{21}$$

with y_R given by Eq.(7) for a decay to two pions. Finally $\tilde{f}_R(K)$ comes from Eq.(10) after having normalized $\tilde{f}_R(K) = C(M^2 - 4m_T^2)^{-1/2}$ by the condition

$$\int \tilde{f}_R(K) d\phi p_T dp_T = 1$$

In this way we find

$$\tilde{f}_R(K) = \frac{2}{\pi} \frac{1}{\sqrt{M^2 - 4m^2}} \frac{1}{\sqrt{M^2 - 4m_T^2}} \tag{22}$$

for the equal mass case.

For the unequal mass case we find in the same way

$$\tilde{f}_R(K) = \frac{1}{2\pi} \frac{1}{k} \frac{1}{\sqrt{k^2 - p_T^2}} \tag{23}$$

where k is given by Eq.(11). Functions $\tilde{f}_R(K)$ are proportional to the probability that a resonance decay leads to a pion with 4-momentum $K = (k_1 + k_2)/2$, see Eq.(2).

Formation time of resonances corresponds to a process in which resonances are - in the statistical average- produced along the boost invariant curve given by

$$\tau_f^2 = t^2 - z^2 \tag{24}$$

In a more realistic model one might think about resonances produced by freeze-out of a thermalized system. The time τ_f in our model mimics the proper time of the freeze-out, but our model does not contain the thermal distribution of resonance momenta within the system at the freeze-out.

Contribution of directly produced pions

In an inside-outside cascade model with hydrodynamical evolution and and with thermalized matter decoupling at (t,z) given by Eq.(24) it is easy to treat directly produced pions and resonances on an equal basis. Both are produced according to Bose - Einstein, or in some approximation, Boltzmann distribution, and after decays of resonances one can calculate the correlation function $C_2(q, K)$.

On the other hand it is not clear whether the hydrodynamical concepts are applicable to a hadronic collision. In our simple model we shall treat direct pions and their contribution to the correlation function in the same way as that of resonances, taking direct pions as products of decay of a resonance with a vanishing life-time. Such a treatment may provide at least some feeling of what may be the effects of directly produced pions.

The life-time of a resonance $\tau_d \approx \hbar/,$ is approaching zero when , is increasing. A resonance with a large width thus corresponds to vanishing τ_d and z_d in Eq.(17). Formation time is taken as equal to that of all other resonances. Taking a large width amounts to integrating over massses of the resonance with a Breit - Wigner distribution. For simplicity we shall take here only a single value of mass. An object with a large width is similar to the behaviour of the S-wave, isospin zero phase shift in $\pi\pi$ scattering. In such a situation we need to add one line to Table 1. Taking the mass of the l=0, I=0 $\pi\pi$ resonance as equal to that of the ρ-meson we get the following parameters

$$y_R = 1.67, \qquad sinh(y_R) = 2.56, \qquad \tau_d = 0, \qquad z_d = 0,$$

$$z_f = 2.56\tau_f, \qquad \tilde{f}_R(K_T = 0) = 0.618, \qquad BR(\sigma) = 1$$

Rapidity density of "σ" production is chosen in such a way as to obtain the desired fraction r_{dir} of direct pions.That means that $w_\sigma(K)$ entering Eq.(19) is determined by the condition

$$r_{dir} = \frac{direct\ \ pions}{all\ \ pions} = \frac{w_\sigma(K)}{w_\sigma(K) + \sum_R w_R(K)}$$

where the sum over R includes other resonances.The calculation then proceeds as above according to Eq.(19).

3 Correlations of two identical pions

In this Section we shall calculate the correlation function $C_2(q, K)$ for identical pions in our model and compare the results with data.The calculation contains two free parameters: the formation time τ_f and the ratio r_{dir} of directly produced pions to all pions in the final state.

Calculation of the correlation function proceeds via Eq.(20) where $\tilde{P}(q; y_R)$ is given by Eq.(19) and $w_R(K)$ by Eqs.(21) and (22) or (23) depending on whether resonance decays to two pions or to a pion and another particle.

In π^+p interactions at 16 GeV [24] the authors have identified meson resonances η, ω, ρ^0 and f_2. Relative contributions of different resonances were found to be strongly p_T-dependent; pions from η- and ω-decays populating mostly the low p_T region, those from ρ and f_2 decays dominating at higher p_T. In the low p_T region it seems that

$$\rho^0 : \omega : \eta : f_2 \approx 0.2 : 0.2 : 0.05 : 0.03$$

as ratios of fractions of the total π^- yield.

In pp interactions at 400 GeV/c about a half of pions is estimated to be produced directly (see Table 9 of Ref.[25]). Resonances, most important for pion production in the region $x_F \geq 0.1$ have inclusive cross-sections of the following unnormalized ratios (see Table 6 of Ref.[25]):

$$< \rho >: \omega : f_2 :< K^* >: \Phi \approx 14 : 13 : 3 : 3,5 : 0.6 \tag{25}$$

where $< \rho >$ denotes averaging over three charged states and $< K^* >$ over four of them.

In pp collisions [26] at CERN - ISR with $\sqrt{s} = 52.5\, GeV$, inclusive production of some of vector and tensor mesons has been measured. Results are consistent with extrapolations of data from lower energies and the fraction of pions and kaons due to decays of resonances has been estimated to be larger than 0.55. Refs.[24-26] contain rather complete lists of papers in which resonance production in hadronic collisions has been studied. Patterns of data in different experiments are qualitatively similar and roughly consistent with expectations based on quark-recombination models [27,28] or Lund Fritiof model [29].

We shall now proceed to calculations of the correlation function $C_2(q, K)$. We would like to stress that it is not our aim to get accurate quantitative results. This is hardly possible at least for two reasons: first - our model is rather simplified and second - knowledge of resonance production in hadronic collisions is not complete. We would rather like to gain a qualitative insight into the question of whether a sum of resonance decay contributions and of direct pions can explain the observed correlations of identical pions and how the correlation patterns depend on the value of the resonance formation time τ_f and on the ratio r_{dir} of direct to all pions. To start with we have to fix some parameters entering the calculations. We shall take the 4-vector K in Eq.(2) as corresponding to $p_T \approx 0$ and $y \approx 0$ in the c.m.s. of hadronic collision. Rapidity y_R of a resonance of mass M decaying to two pions is then given by Eqs.(7) or (12), where transverse mass reduces to the pion mass. We shall treat three-body decays $\omega \to 3\pi$ and $\eta \to 3\pi$ as two- body decays $\omega \to \pi d$ and $\eta \to \pi d$ with "d" denoting a "dipion". The mass m_d in the ω-decay is taken as $m_d = m_d(\omega) = 470\, MeV$ and $m_d(\eta) = 350\, MeV$ what corresponds to symmetric decay kinematics. In this case rapidity of a resonance decaying to a pion with $y = 0$ and small p_T is given by Eq.(12).

$$y_R = ln(\alpha + \sqrt{\alpha^2 - 1}), \qquad \alpha = \frac{M^2 - (m_d^2 - m^2)}{2mM} \tag{26}$$

This expression is valid also for the decay $K^* \to K\pi$. All parameters entering our calculation of $C_2(q, K)$ via Eq.(19) are given in Table 1, which contains in the last row also parameters concerning directly produced pions. We shall briefly recapitulate symbols in Tab.1 and relations defining them: y_R is the rapidity shift between a resonance and its decay product, see Eqs.(7) and (12) for equal resp. unequal mass cases,$\tau_d = 1/$, where , is the resonance width, $z_d = sinh(y_R)\tau_d$ is the mean decay distance; $z_f = sinh(y_R)\tau_f$ where τ_f is the formation time of a resonance; $\tilde{f}_R(K)$ is a kinematical factor proportional to the probability density of producing a pion with a given K in the resonance decay. Branching ratio $BR(R)$ is recalculated to an average charge state of the resonance. For instance in the case of the ρ meson we have three charged states.We assume that in the central rapidity region

$$\frac{dn(\rho^+)}{dy} \approx \frac{dn(\rho^0)}{dy} \approx \frac{dn(\rho^-)}{dy} \approx \frac{dn_\rho}{dy} \tag{27}$$

In the sum over ρ^+, ρ^0 and ρ^- decays we shall have $2\pi^- + 2\pi^0 + 2\pi^+$. For $dn_\rho/dy = 1$ we shall thus have two like-sign pions. This factor is included into $BR(\rho)$. In the column Adn_R/dy we give non-normalized ratios of central rapidity density which are guessed from data of Ref.[25]. The symbol dn_R/dy denotes rapidity density averaged over charged states of resonances in the spirit of Eq.(27). The correlation function is then given by Eqs.(19-21).

According to Eq.(20) $C_2(q,k)$ is a weighted sum of contributions of individual resonances. To see that resonances and direct pions give quite different contributions we present in Fig.2 correlation functions corresponding to the assumption that all pions are decay products of a particular resonance - the weight $w_R(K)$ of this resonance is 1 and all other weights vanish. The contribution of direct pions is calculated in the same way and also presented.

In the same Fig.2 we plot also the data of EHS/NA-22 Collaboration given in Fig.5b of Ref.[16]. The data correspond to averaging over transverse momenta $0 < Q_T < 40\,MeV/c$ and this narrow interval permits us to compare our calculations done for small transverse momenta with this data.

The interpretation of Figs. 2a, 2b and 2c is rather simple. In Fig.2a corresponding to $\tau_f = 0.2\,fm/c$ direct pions are originated by decay of a resonance with formation time of 0.2 fm/c and vanishing mean life-time for the decay. Because of that direct pions are created within a short distance from the collision point and the Fourier Transform of this density of sources is rather broad in q. A typical value of q for directly produced pions is $\hbar/sinh(y_R)\tau_f \approx 0.4$ GeV/c. For resonances like ρ, Δ, f_2, K^* characteristic time is increased by their decay time, for $\tau_d \approx 1.3$ fm/c, corresponding to a resonance width of about 150 MeV, typical longitudinal momentum is $\hbar/sinh(y_R)\tau_d \approx 60$ MeV/c.

With increasing formation time both resonance contribution and that of direct pions become steeper in q. For τ_f equal to 0.2 or 0.4 fm/c the contribution of direct pions decreases slower than the data so that a cocktail composed of resonance decay products and of direct pions has a chance to describe the data,although at the price of increasing r_{dir} for increasing τ_f.

For $\tau_f \approx$1 fm/c even the contribution of direct pions decreases faster than the data and any cocktail composed of resonance decay products and direct pions is bound to fail.

In Fig.3 we show the dependence of $C_2(q)$ on both τ_f and r_{dir}.As can be seen in Fig.3a a reasonable qualitative agreement with data is obtained for $\tau_f \approx 0.2$ fm/c and $r_{dir} \approx$ 0.50 - 0.6. For $\tau_f \approx$0.4fm/c the agreement can be reached with $r_{dir} \approx$0.7 which seems to be excluded by data on resonance production preferring a lower fraction of directly produced pions.

In Figs.2 and 3 our curves are somewhat below the point with the lowest value of q.This might be due to the fact that we have not included the contribution of η.For the study of this point one would need to take into the account also details of the binnig procedure.

In our simplified model transverse momenta of resonances are put equal to zero. With transverse momenta of resonances included we expect that formation times τ_f required by the data will slightly increase, since resonances moving not exactly along the z-axis would need more time to decay in the region with the same longitudinal dimension.

4 Formation time of hadrons and the evolution of pA and AB interactions

In the present section we shall discuss the relationship between the value of the formation time and the development of cascades in pA and AB interactions. We assume that in a pA interaction the incoming proton interacts with ν nucleons in the nucleus A. Between two subsequent nucleon-nucleon subcollisions only hadrons with (Lorentz dilated) formation time smaller than time interval between two sub-collisions.

We assume that in the cms frame of a nucleon-nucleon subcollision the fragmentation time of a hadron is given by the following expression

$$t_f = \tau_f cosh(y) \tag{28}$$

where y is the rapidity in this system and $cosh(y)$ is the usual factor of Lorentz time-dilation. We shall take the following form for the probability that a hadron (resonance or directly produced pion) with rapidity y has been formed in time t after the collision

$$P(t;y) = 1 - e^{-t/t_f} \tag{29}$$

The mean time between two subsequent collisions of a given nucleon is

$$t_0 = \frac{\lambda}{2\gamma c} \tag{30}$$

where γ is the Lorentz contraction factor, $\gamma \approx 10$ at the CERN SPS and λ is the mean free path for a nucleon in the nucleus at rest. The factor 2c in the denominator the relative velocity of the two colliding nucleons.A typical value of t_0 is thus about 0.15 fm/c. The time intervals between the subsequent subcollisions are Poisson distributed with the mean value t_0 and the averaged probability that a hadron with rapidity y is formed between two subcollisions becomes

$$P(y) = \int_0^\infty (1 - e^{(-t/t_0)}) \frac{1}{t_0} e^{-t/t_0} dt = 1 - \frac{t_f}{t_f + t_0} = 1 - \frac{\tau_f cosh(y)}{t_0 + \tau_f cosh(y)} \tag{31}$$

The expression for P(y) depends only on the ratio τ_f/t_0 which can be estimated only very roughly as follows. The value of t_0 is about 0.15.

Our analysis of EHS/NA- 22 Collaboration data has given the value of τ_f as $0.2 \leq \tau - f \leq 0.4$. As a very crude estimate we thus obtain for the ratio $\alpha = t_0/\tau_f$ values in the interval 0.5 to 1. The resulting probability

$$P(y) = 1 - \frac{cosh(y)}{\alpha + cosh(y)} \tag{32}$$

gives the production of hadrons in a given subcollision. For $\alpha = 0$ we get P(y)=0 as expected and the picture of subcollisions goes over to that of the wounded nucleon model with full fragmentation occuring only after the last subcollision.

5 Comments and Conclusions

We have described above a very simplified model of effects caused by resonance formation and decay on Bose - Einstein correlations of identical pions in hadronic collisions. Due to the simplicity of the model our results should rather be considered as hints to what one can expect in more realistic calculations. In particular our treatment of direct pions is rather model dependent. If it would turn, for instance, that direct pions are produced faster than in our model, their contribution to $C_2(q)$ would be broader and to get the observed shape of $C_2(q)$ the resonance contribution to $C_2(q)$ should be narrower what would mean longer τ_f.

In our model we have assumed that all resonances are produced from the same point (t,z)= (0,0). In a more realistic calculation the resonances should be produced from a space-time region with longitudinal radius of about R/γ where R is the nucleon radius and γ is the Lorentz contraction factor, for the SPS CERN energy region $\gamma \approx 10$. Taking this into account our correlation function $C_2(q, K)$ would be multiplied by the FT of the density distribution of sources of resonances, what would slightly decrease the resulting values of τ_f.

With this reservation we can summarise our results.

- The correlation function $C_2(q, K)$ for $\vec{K} \approx 0$ as measured by the EHS/NA-22 Collaboration [16] in π^+ interactions at 250 GeV/c can be understood as due to an interplay of resonance decays and of directly produced pions provided that the fraction of directly produced pions $r_{dir} \approx 0.5$ and the formation time of resonances and direct pions is rather short $\tau_f \approx 0.2$ fm/c. For the formation time of $\tau_f \approx 0.4$ fm/c the fraction r_{dir} increases to about 0.7 and for $\tau_f \approx 1$ fm/c consistency with data cannot be achieved.
- Note that our estimate of the fraction of directly produced pions is larger than results obtained by Lednický and Progulova [23].
- Our simple model shows in a very transparent way a strong dependence of the correlation function $C_2(q, K)$ on the value of K=$(k_1 + k_2)/2$ and in particular on the average transverse momentum K_T of the two identical pions.
- In our model resonance formation and decay plays an important role and as a consequence of that the correlation function $C_2(q)$ is quite different from a Gaussian. This indicates that the data on correlations in hadronic collisions should be rather fitted by functions which correspond to a sum of directly produced pions and one or two resonances.When taking only one resonance one should probably take parameters of the ρ to take into account resonances of width comparable to that of the ρ and when taking also a second resonance one could take parameters of the ω to take into account also objects with a longer life-time.

Models analyzing effects of resonance formation and decay on correlations of identical particles have been studied earlier by numerous authors [30- 40]. Our model is also presented in [41]. Conclusions about resonance formation times and average life-times have been made by Lednický and Progulova [23] who have considered a model containing ρ-mesons and direct pions, by Csörgö et al. [34] who have evaluated analytically the average formation time of resonances as 0.77 ± 0.1 fm/c and mean life-time of resonances as 2.88 fm/c and used then the Monte Carlo program SPACER to analyze data on Si+Au collisions at 14.5GeV per nucleon and O+Au interactions at 200 GeV/nucleon.

Padula and Gyulassy [36-38] have analyzed pp and $\bar{p}$p data at CERN ISR energies and in particular the sensibility of data to the abundance of resonances. They have found that the data are inconsistent with the full resonance fractions as predicted by the Lund model.Their results are consistent with those of Kulka and Lörstad [40] and with our results at lower energies as shown in Fig.2 above. The reason of this result is due to to the fact that resonances tend to increase R_L whereas direct pions work in the opposite direction.

In most of analyses the presence of resonances leads to marked deviations from Gaussian shapes of the correlation function $C_2(q)$, reasons for that being simply visible in our model.

It would be most interesting to have data on correlation functions for pp, pA, and AB collisions at the same energy which would permit to study differences of correlation functions as a function of the atomic number of colliding particles and search for the onset of collective expansion, which should be visible via long time delays [42-45].Unfortunately the increase of $\ll z^2 \gg$ may be due both to an increase of the time delay and to the increase of the abundance of resonances and these two mechanisms should be disentangled before firm conclusions could be done.A step in this direction has been recently performed by Wiedemann [46] in an interesting analysis which combines hydrodynamics in heavy-ion collisions with effects of resonance decays.

There is a lot of most interesting aspects of data which we have not discussed in the present paper. Apart of the p_T-dependence of correlation function these include at least: multiparticle correlations and intermittency, correlations of unlike pions which appear naturally in models based on resonance decays, and the rapidity dependence of correlation functions.We have also limited ourselves to a simple situation with two identical pions having the same energy and have studied only the dependence of the correlation function on the difference of the longitudinal momenta Q_L. The model can be generalized also to other types of variables upon which the correlation function depends and we hope to return to these issues in the near future.

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Figure Captions

Fig.1 Two interfering amplitudes for production of identical pions with momenta $\vec{k}_1$ and $\vec{k}_2$.

Fig.2 Contributions of individual resonances and of directly produced pions to the correlation function $C_2(q)$. Data points taken from Fig.3b of Ref.[31]. Contributions are plotted for three values of the formation time: a) τ_f= 0.2 fm/c; τ_f = 0.4 fm/c; c) τ_f = 1 fm/c.

Fig.3 Comparison of data [31] on $C_2(q)$ for $0 < p_T < 0.04$ GeV/c with coctails of resonances and directly produced pions: a) τ_f = 0.2 fm/c; b)τ_f = 0.4 fm/c.

Table 1: Table 1. Basic parameters for calculations of identical pions (lengths in GeV^{-1})

$Res.$	y_R	$sh(y_r)$	τ_d	z_d	z_f	$f_R(K)$	$BR(R)$	$A\frac{dn_r}{dy}$	$w(r)$
ρ	1.67	2.56	6.66	16.96	$2.56\tau_f$	0.618	2	0.31	0.38
ω	1.257	1.615	118.6	191.6	$1.615\tau_f$	1.615	0.89	0.31	0.45
f_2	2.35	5.2	5.41	28.1	$5.2\tau_f$	0.21	0.57	0.07	0.01
K^*	1.47	2.06	41.2	84.9	$2.06\tau_f$	0.96	1.33	0.08	0.1
Δ	1.26	1.62	8.3	13.5	$1.62\tau_f$	1.57	1.33	0.11	0.23

Table 2: Table 2. Dependence $y_R = y_R(K_T)$ for a selected set of resonances

$K_T[GeV/c]$ Resonance	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
ρ	1.67	1.62	1.45	1.24	1.03	0.80	0.56	0.205	
f_2	2.19	2.14	1.98	1.80	1.61	1.43	1.27	1.11	0.96
K_*	1.47	1.41	1.23	1.01	0.77	0.34			
Δ	1.25	1.19	0.99	0.74	0.40				
ω	1.24	1.18	0.98	0.73	0.38				
η	0.76	0.67	0.31						

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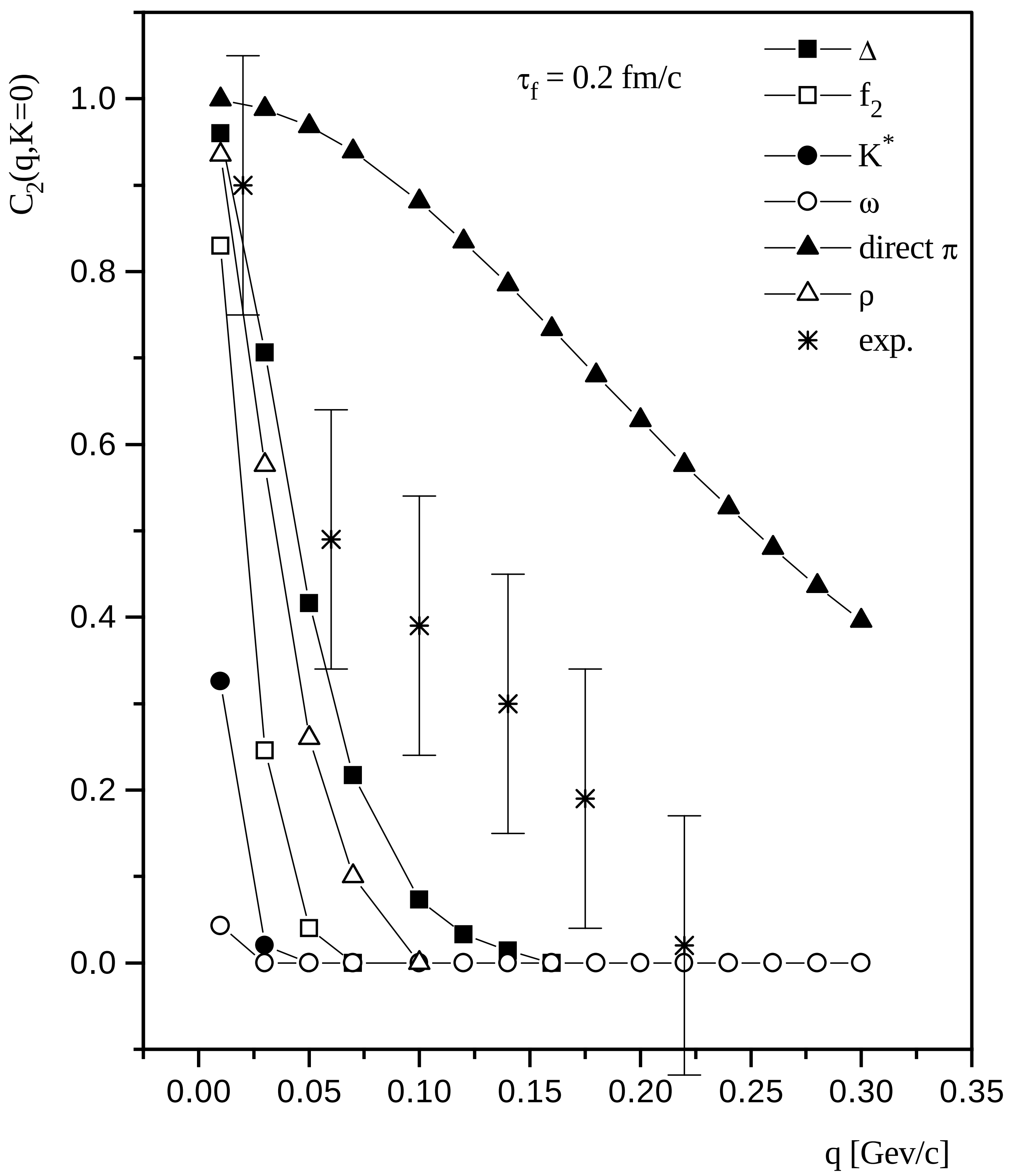

τf = 0.2 fm/c
Δ
f2
K*
ω
direct π
ρ
exp.
C2(q,K=0)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
q [Gev/c]

Fig.2a